RESEARCH STATEMENT

PHILIPP SCHLICHT

My research contributes to the interactions between set theory, logic and other mathematical fields. The infinite is at the heart of most mathematical research and as always fascinated me the most. Can we work with infinite objects just like with finite ones and why? Why do strange infinite objects exist that have no counterpart in the finite world? Why is the study of infinity useful to study finite objects?

Set theory. I study set theory and infinite combinatorics. These research fields ask in which circumstances uncountable objects and structures behave like finite or countable ones. There are many surprising phenomena in the uncountable such as the Banach-Tarski paradox that violates our intuition about size. Set theory can be very broadly divided into (a) infinite combinatorics, the study of the combinatorics of uncountable sets and new phenomena of the uncountable, and (b) descriptive set theory, the study of more concrete mathematical objects whose properties are closer to that of countable sets. Lebesgue and Borel introduced descriptive set theory with the aim to study concrete subsets of the reals and of function spaces and show that they have nice properties. Descriptive set theory tries to classify objects that appear in set theory, analysis, measure theory and other fields according to their complexity and study their properties. The hierarchy of complexity begins with open, closed and Borel sets. For example, one can ask about the complexity of the set of all differentiable functions in $C[0,1]$ and whether all sets of reals of a given complexity are Lebesgue measurable.

The use of **graph-theoretic** methods is an important development in descriptive set theory in the last decade. One can translate any problem about finite and countably infinite graphs to the context of graphs on the reals and other spaces. This approach was initiated by Kechris, Solecki and Todorčević [KST99] and pursued intensively by Ben Miller [Mil12]. It has led to various applications of ideas from graph theory and computer science [MU17, GR21]. My current research project funded by the EPSRC focuses on this area and includes the highlight:

- Extending a structure theorem for infinite-dimensional hypergraphs to large spaces [SS23]. This graph-theoretic approach allows us to solve several problems in the descriptive set theory of generalised Baire spaces that resist attempts to lift classical proofs.

Descriptive set theory provides powerful tools for the **classification** of many kinds of mathematical objects. In particular, one can often understand a class of algebraic objects such as groups as elements of a natural metric space and measure the complexity of the isomorphism relation for this class. A recent highlight is:

- Showing that the complexity of the isomorphism problem for oligomorphic groups is much simpler than conjectured by Kechris, Nies and Tent [NST22]. These groups naturally appear as automorphism groups in model theory.

Descriptive set theory is closely connected with **real analysis** and **measure theory**, since it arose from these fields. I recently obtained the following applications in these areas:

- Analysing ranks in real analysis that measure the complexity of an object by an ordinal number using techniques from descriptive set theory [CSW22] and thereby extending work of Kechris, Marker and Sami [KMS89].
- Investigating generalisations of Lebesgue's density theorem from measure theory by replacing the σ-ideal of sets of measure 0 by arbitrary σ-ideals [MSSW22]. A main result shows that no such generalisation is possible for the ideal of countable sets.

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Lebesgue measurability and the Baire property of subsets of $\mathbb{R}^n$ can be understood as a form of symmetry. My study of other such properties includes:

- Extending the perfect set property [Sch17] and the Hurewicz dichotomy [LRS16] to generalised Baire spaces. These results are surprising since many basic results in classical descriptive set theory do not generalise.
- Developing a framework analogous to countably based complete metric spaces in the setting of large spaces [CS16, ARS21]. Here the role of completeness is replaced by the existence of winning strategies in topological games.

Logic comes into play in most of my research because of the tight connections between the complexity of sets and the **logical strength** of axiom systems that suffice to prove properties of sets at this complexity. This becomes relevant just beyond Borel sets and their projections, the analytic sets, on the first level of the projective hierarchy. This topic connects to the theory of infinite games, in particular the existence of winning strategies (determinacy). The following are highlights in this area:

- Extending Hjorth's analysis of equivalence relations from the second to all even levels of the projective hierarchy using determinacy, large cardinals and inner model theory [Sch14].
- Initiating the study of the connection between long infinite open games with σ-projective determinacy [AMS21].
- Proving the preservation of projective determinacy in extensions for classical forcings [CS21] and very general results for iterated forcings [SSS23], extending work of Goldstern, Judah and Shelah on a technique showing that some iterated forcing constructions preserve the Baire property [IS88, GJ92].

Properties of sets beyond Borel and analytic often cannot be decided by the standard set theoretic axioms. Cohen's **forcing** technique is used to construct models of the axioms of set theory that witness the independence of a statement from the axioms of set theory. A highlight from my study of a global form of this technique called class forcing is:

- Proving the failure of the forcing theorem for class forcing [HKL$^+$16] and describing its consistency strength in second order set theory [GHH$^+$20]. The forcing theorem is the most fundamental result about set forcing, saying that statements true in the corresponding forcing extensions are forced and forced statements are true.

Ideas from **computability** often appear in descriptive set theory in the form of algorithms. I apply descriptive set theory to the Wadge hierarchy that is studied in both set theory and in theoretical computer science and to notions of computation beyond the Turing barrier. My research includes:

- Showing that the Wadge hierarchy on the reals and other separable metric spaces of positive dimension has a completely different structure than the classical Wadge hierachy [RSS15, Sch18, IST19].
- Proving that higher randomness at the level of Hamkins and Kidder's infinite time Turing machines is a well-behaved form of algorithmic randomness [CS17, CS18], extending Hjorth and Nies's study of randomness at the level of analytic sets.

Computability. Computability is the study of functions on the natural numbers and on the Cantor space of infinite bit sequences that can be computed by a Turing machine. In a similiar way as concretely defined sets in descriptive set theory have good properties, this is true to a much stronger extent in structures presented by finite automata (automatic structures). These have been of strong interest in computer science, since all their properties are decidable by algorithms (although not efficiently). In mathematics, such structures have appeared in geometric group theory in the form of automatic finitely presented groups. A popular math application to infinite chess appeared in my paper [BHS12]. My research in this area includes:

- Extending the properties of automatic structures to those defined on ordinals [SS13, HKS17] and connecting these to tree-automatic structures [JKSS16, JKSS19].

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